The bivariate J-function to analyse positive association between galaxies and galaxy filaments

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Abstract. The compelling and intrinsic network of galaxies builds up a complex structure for the Universe. In this work positive association between long bridging structures named galaxy filaments and a photometric redshift galaxy dataset is under investigation. Possible positive association is studied by the use of a bivariate J-function.

Keywords. Methods: statistical – galaxies: statistics – large-scale structure of Universe

1 Introduction

Large scale galaxy surveys show the intrinsic structure of the Universe: bridging vast filamentary patterns, galaxy clusters, sheets, superclusters and immense regions almost devoid of galaxies named voids (Jõeveer et al. (1978)). These maps are dominated by galaxy filaments, which connect the structures into a web (Pimbblet et al. (2005)). A mathematical framework named the Bisous model (Stoica et al. (2010)) has been applied on observed large scale galaxy datasets in the paper Tempel et al. (2014). In the latter paper they estimated the filamentary pattern from Sloan Digital Sky Survey (York et al. (2010)) (SDSS) spectroscopic galaxy data (these galaxies have precise distances in redshift[1] space).

In this study we analyse whether these galaxy filaments (Tempel et al. (2014)) are positively associated with the Sloan Digital Sky Survey photometric redshift galaxies dataset (these galaxies have rather uncertain distance estimations in redshift space) compiled in (Beck et al. (2016)). The galaxies can be viewed as points in 3-dimensional Euclidean space (Martinez & Saar (2001)). The Cartesian coordinates of these galaxies are point locations in 3−dimensional Euclidean space and other properties can be considered as marks. Thus the galaxies situated in a survey region can be seen as a realization of a marked point process in a compact set.

[1] Characteristic that estimates the measure of distance in cosmology.

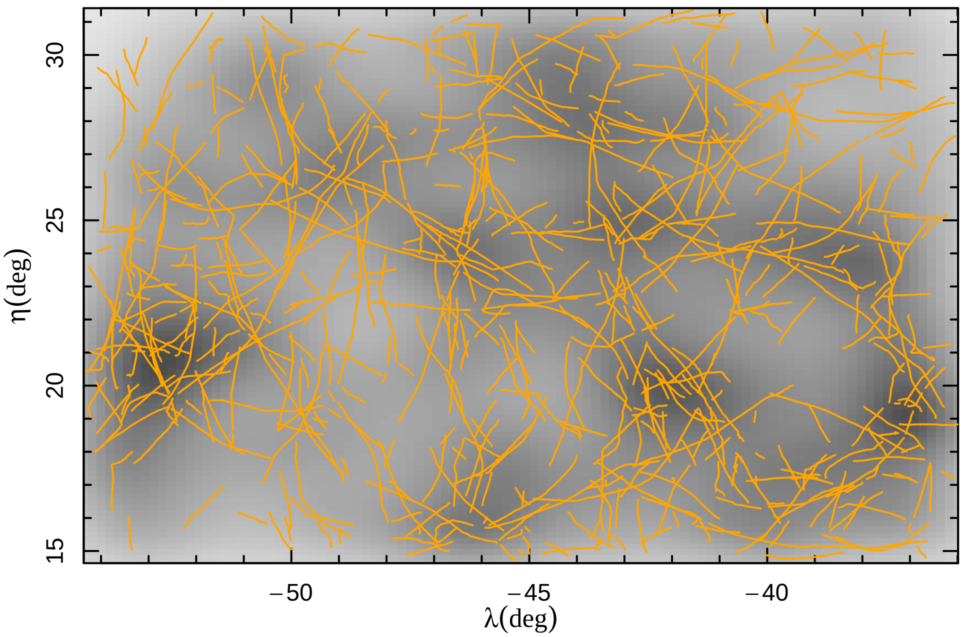

Figure 1: Visualization of the two datasets in spherical sky-coordinates: kernel smoothed 2-dimensional density from the distribution of photometric galaxies on the viewed region of the sphere (grey background density) and the filamentary spines (orange spines). All the objects locate in the distance range of $200 - 400$ Mpc and the darker grey denotes a higher density of photometric galaxies.

2 Data

In this section the analysed photometric redshift galaxies dataset and the filamentary pattern catalogue are characterized. Limitations will be set on all of the observed objects dependent on their distance to the observer.

Figure 1 depicts the two studied datasets in spherical sky coordinates η and λ. From visual inspection one can notice that the higher density of filamentary spines seems to follow the higher density of photometric redshift galaxies. Further on this possible clustering effect is analysed with the use of the bivariate $J-$function.

2.1 Filamentary network

The filamentary spines catalogue holds information about 46 403 objects. They form a random set of objects $Y_{\rm fil}$ in the 3−dimensional Euclidean space. The mapping of a point in R^3 to the spherical coordinates latitude η and longitude λ is done using

$$\begin{aligned}\eta &= 2\arctan\left(\frac{y}{\sqrt{x^2+y^2}}-x\right),\\ \lambda &= \arcsin\left(\frac{z}{\sqrt{(x^2+y^2+z^2)}}\right).\end{aligned} \tag{1}$$

In paper (Kruuse et al. submitted) the mean distance used to characterise spine Y distance to the observer O is defined as:

$$\bar{d}(O,Y)=\frac{1}{l(Y)}\int\limits_{Y} d(O,y)\mathrm{d}y, \tag{2}$$

where $l(Y)$ is the associated spine length and $d(O,y)$ is the distance from the observer O to the point y of the spine Y. The integral in (2) is computed along the considered spine. These mean distances will be used to limit the filamentary spines into the distance range of $200-400$ Mpc. Filamentary spines, which satisfy this condition, are further analysed. The dataset Y_{fil} will be of 20 367 filamentary spines. The chosen distance range carries the bulk of the filamentary spines.

2.2 Photometric galaxies

The rather uncertain redshift z_{photo} estimations ($\delta z_{\mathrm{photo}} \leq 0.05$) in the catalogue (Beck (2005)) are used to limit the galaxies into the distance range of $200-400$ Mpc. Which leaves us with the dataset of 236 850 photometric galaxies. Their locations are defined on the sphere by their radian latitude η and longitude λ. They are viewed as a configuration of points X_{photo} in a sphere S^2.

3 The bivariate $J-$function

The datasets studied are fit for being analysed with spatial statistics tools (Baddeley et al. (2015); Martinez & Saar (2001)). One can consider the photometric galaxies set like a point processes realization and the filamentary set as a realization of a random set. The bivariate $J-$function can be used to study possible positive association between these objects.

Thorough mathematical description of the theory of point processes and probabilistic models therein are written in van Lieshout (2000), Møller & Waagepetersen (2004), Illian et al. (2008), Chiu et al. (2013) and Baddeley et al. (2015). Here we will present the border corrected estimations for the empty space function, nearest-neighbour function and the bivariate $J-$function as they are described in van Lieshout & Baddeley (1996), Foxall & Baddeley (2002) and Baddeley et al. (2015). All the described summary statistics have exact formulas for the homogeneous Poisson point process, which represents a completely random pattern. A rigorous representation of the Poisson point process is written Baddeley et al. (2015).

3.1 Empty space function

First of all the shortest distances from a point $w \in W$ to a subset $A \subset W$ is noted by the following form

$$d(w, A) = \inf_{a \in A} \|w - a\|.$$

The border corrected estimator for the empty space function from any arbitrary point w_i to the random set of Y is of the following form

$$\widehat{F}_Y(r) = \frac{\sum_i \mathbb{1}\{d(w_i, W^c) \geq r\} \mathbb{1}\{d(w_i, Y) \leq r\}}{\sum_i \mathbb{1}\{d(w_i, W^c) \geq r\}} \tag{3}$$

with W^c the border of W and $\{w_i, i = 1, 2, \ldots\}$ a finite family of arbitrary points in W. The estimation $\widehat{F}$ is a cumulative distribution function of smallest distances from arbitrary points to the closest object in the random set.

3.2 Nearest-neighbour function

The estimator of the nearest neighbour distribution from any point in X to the random set Y is of the following form

$$\widehat{G}_{X,Y}(r) = \frac{\sum_i \mathbb{1}\{d(x_i, W^c) \geq r\} \mathbb{1}\{d(x_i, Y) \leq r\}}{\sum_i \mathbb{1}\{d(x_i, W^c) \geq r\}}, \tag{4}$$

where $\{x_i, i = 1, \ldots\}$ is the observed finite point configuration of X. The nearest-neighbour function is a cumulative distribution function of smallest distances from a point of the point process X to the nearest object in the random set Y. The estimation of $\widehat{G}$ measures association between the objects of type X and type Y. If the point process X and the random set of objects Y are independent, then $\widehat{G}_{X,Y} \equiv \widehat{F}_X$.

3.3 Bivariate $J-$function

The bivariate J function is obtained by the nearest-neighbour function $G_{X,Y}(r)$ and empty space function $F_Y(r)$ in the following form:

$$\widehat{J}_{X,Y}(r) = \frac{1 - \widehat{G}_{X,Y}(r)}{1 - \widehat{F}_Y(r)}. \tag{5}$$

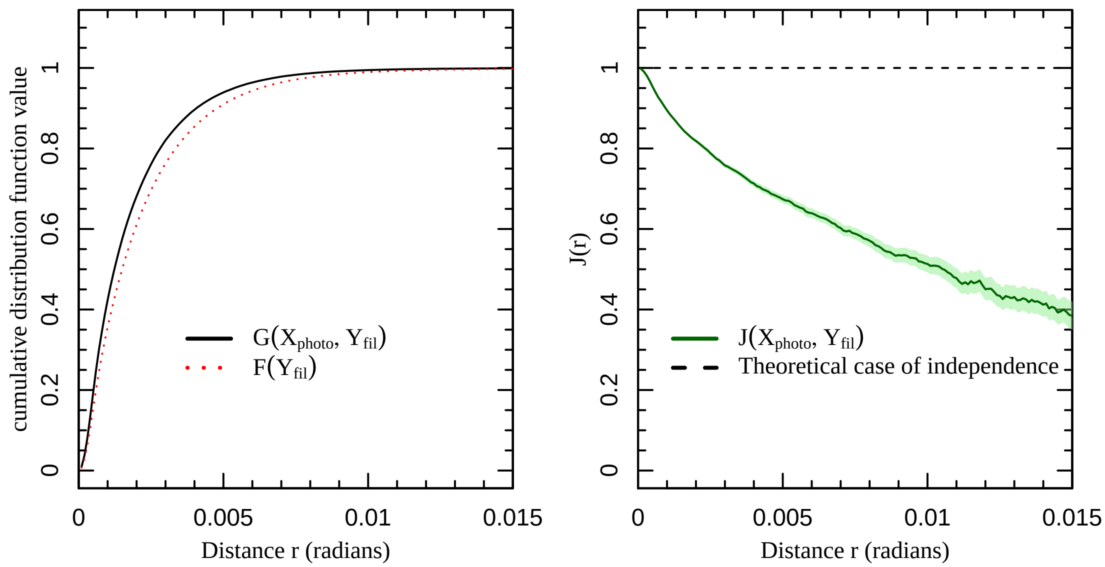

Figure 2: On the left panel: the results of the empty space function $\widehat{F}_{\mathrm{Y_{fil}}}$ (red dotted line) and nearest-neighbour function $\widehat{G}_{X_{\mathrm{photo}},Y_{\mathrm{fil}}}(r)$ (black continuous line). On the right panel: the results of the bivariate $J-$function $\widehat{J}_{\mathrm{X_{photo},Y_{fil}}}$ with the 0.95−confidence interval. The theoretical case of independence is noted by the dashed dark line.

The $J-$function measures association between X and Y (Foxall & Baddeley 2002). If X and Y are independent, then $J_{X,Y} = 1$. Values of $J_{X,Y}$ close to 1 suggest independence, values higher than 1 suggest negative association and values lower than 1 suggest positive association.

4 Applying the bivariate J-function

In this section we will present the results obtained to analyse possible positive association between the photometric galaxies X_{photo} and filamentary spines Y_{fil}, which locate at the distance range of $200 - 400$ Mpc from the observer. The border corrected empty space function $\widehat{F}_{Y_{\mathrm{fil}}}$, border corrected nearest-neighbour function $\widehat{G}_{X_{\mathrm{photo}},Y_{\mathrm{fil}}}(r)$ and the bivariate $J-$ function $\widehat{J}_{\mathrm{X_{photo},Y_{fil}}}$ are presented.

Figure 2 left-hand panel represents the results for the border corrected estimations of the G and F statistics. The values for the empty space function are below of the values of the nearest neighbour function for small values of distance r. To investigate whether positive association between the photometric galaxies and filamentary spines exists the bivariate $J-$function is presented in the Fig. 2 right-hand panel.

Figure 2 right-hand panel draws the result for the bivariate $J-$function (eq. (5)). The decreasing $\widehat{J}_{\mathrm{X_{photo},Y_{fil}}}$ values below the theoretical reference case (which would indicate independence between observed sets) represent positive association between the photometric galaxies and all filamentary spines. Indicating that the photometric galaxies carry information about the filamentary network, which has been detected from the spatial distribution of spectroscopic galaxies.

5 Conclusion

The bivariate $J-$function is well fit to study possible positive association between complex filamentary structures and galaxies, which can be described as a realisation of a point process. The statistical signal of positive association indicates that the photometric galaxies hold information about the location of the filamentary network. The result obtained from the bivariate $J-$function encourages to use the photometric galaxies. They would increase the number density of galaxies per volume of space, which will contribute to a more detailed analysis of the cosmic web. A thorough analysis with the bivariate $J-$function to study eventual correlation between galaxies and galaxy filaments is done in Kruuse et al. (submitted to A & A).

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